Tutorial 8.

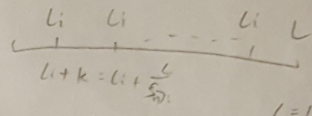
Preliminary:

Sinking-fund method. A loan calls n interest payments Li and pay the single lump sum of amount L at time n.

Li payment in each period

$\frac{L}{s_{\overline{n}|j}}$ is level sinking fund deposit.

Outlay is $L(i+\frac{1}{s_{\overline{n}|j}})$.

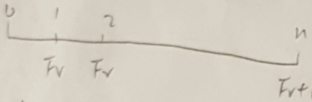


$L = k s_{\overline{n}|j}$

Bond Price: P: price; F: face amount; C: redemption amount

r: coupon rate; j: yield rate.

$P = Cv_j^n + Fr(v_j + v_j^2 + \cdots + v_j^n) = Cv_j^n + Fr\,a_{\overline{n}|j}$.



When $F=C$, $P = Fv_j^n + Fr\,a_{\overline{n}|j} \xrightarrow{v_j^n = 1 - j a_{\overline{n}|j}} F + F(r-j)a_{\overline{n}|j}$.

Exercise:

3.3 6s.

$L=1000$, $n=10$, $i=10\%$, $j=14\%$

$L = P a_{\overline{n}|i} \Rightarrow P = 162.75$.

another strategy.

Sinking fund deposits are $P - L_2 = 162.75 - 1000\times 10\% = 62.75$.

Accumulate Value of sinking fund $AV = 62.75 s_{\overline{n}|j} = 1213$.

Balance $1213 - 1000 = 213$.

three companies 4

financial instruments:

derivative: (options, futures)

equities: (stocks)

debt (bonds, mortgage).

4.1.4.

$r = \frac{6\%}{2} = 3\%$, $j = \frac{5\%}{2} = 2.5\%$, $n = 8\times 2 = 16$, $r' = \frac{5.5\%}{2} = 2.75\%$.

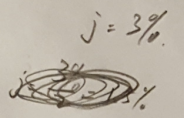
$C = F$. $P = Fv_j^n + Fr a_{\overline{n}|j} = Fv_j^{2m} + Fr' a_{\overline{2m}|j}$

$\Rightarrow v_j^n + r a_{\overline{n}|j} = v_j^{2m} + r' a_{\overline{2m}|j}$.

$\Rightarrow m = 21.5$ years.

4.1.12.

$F_1 = F_2 = 100$, $P_1 + P_2 = 240$, $r_1 = 2r_2$, $P_1 - P_2 = 24$.



$\begin{cases} P_1+P_2 = 240 \\ P_1 - P_2 = 24 \end{cases} \Rightarrow P_1 = 132,\ P_2 = 108.$

$P_1 = F_1 + F_1(r_1 - j)\, a_{\overline{n}|j} \Rightarrow 132 = 100 + 100(2r_2 - j)\, a_{\overline{n}|j}$

$108 = 100 + 100(r_2 - j)\, a_{\overline{n}|j}$

$\Rightarrow \frac{32}{8} = \frac{2r_2 - j}{r_2 - j}$,

$\Rightarrow r_2 = 0.0225$, $r_1 = 0.045$.

4.1.14.

$t = 25\%$, $F = 1000$, $r = 4\%$, $n = 10$, $j = 5\%$.

$P = Cv_j^{10} + Fr a_{\overline{n}|j}$

$\Rightarrow P = 908.78$.

$C = 1000 - 0.25 \times (1000 - P)$

↑ discount